Gymnasium

1. Stegreifaufgabe aus der Mathematik

Klasse 11

- Lösungen -

1. **a)** $\lim\limits_{x\to\infty}\frac{x}{1+x^2} = \lim\limits_{x\to\infty}\left(\frac{1}{\frac{1}{x}+x}\right) = \underline{\underline{0}}$ | $\lim\limits_{x\to\infty}\frac{1}{x} = 0$

b) $\lim\limits_{x\to\infty}\frac{2x-1}{x} = \lim\limits_{x\to\infty}\left(2-\frac{1}{x}\right) = \underline{\underline{2}}$

c) $\lim\limits_{x\to\pm\infty}\frac{2+x}{x^2} = \lim\limits_{x\to\pm\infty}\left(\frac{2}{x^2}+\frac{1}{x}\right) = \underline{\underline{0}}$

d) $\lim\limits_{x\to\pm\infty}\frac{x^5}{x^4+1} = \lim\limits_{x\to\pm\infty}\left(\frac{x}{1+\frac{1}{x^4}}\right) = \underline{\underline{\pm\infty}}$

e) $\lim\limits_{x\to 1}\frac{x^2-1}{x-1} = \lim\limits_{x\to 1}\frac{(x+1)(x-1)}{x-1} = \lim\limits_{x\to 1}(x+1) = \underline{\underline{2}}$

f) $\lim\limits_{x\to -2}\frac{5(x^2-1)}{x+1} = 5\cdot\lim\limits_{x\to -2}\frac{(x+1)(x-1)}{x+1} = 5\cdot\lim\limits_{x\to -2}(x-1) = 5\cdot(-3) = \underline{\underline{-15}}$

g) $\lim\limits_{x\to 0}\frac{x^2+4x-8}{\cos x} = \frac{\lim\limits_{x\to 0}(x^2+4x-8)}{\lim\limits_{x\to 0}\cos x} = \frac{-8}{1} = \underline{\underline{-8}}$

h) $\lim\limits_{x\to\infty}\frac{4x+1}{2x+1} = \lim\limits_{x\to\infty}\frac{2(2x+1)-1}{2x+1} = \lim\limits_{x\to\infty}\left(2-\frac{1}{2x+1}\right) = 2-0 = \underline{\underline{2}}$

i) $\lim\limits_{x\to\infty}\frac{3x^2-1}{x^2-4} = \lim\limits_{x\to\infty}\frac{3(x^2-4)+11}{x^2-4} = \lim\limits_{x\to\infty}\left(3+\frac{11}{x^2-4}\right) = 3+0 = \underline{\underline{3}}$

- Lösungen -

2. geg.: $f(x)=\frac{2x-1}{5x^2+1}\sqrt{3+\sin 2x}$

$$g(x)=\frac{2x-1}{5x^2+1}\cdot 2 \;=\; \frac{4x-2}{5x^2+1}$$

$$\lim\left(g(x)\right)\frac{4x-2}{5x^2+1} \;=\; \frac{\frac{4}{x}-\frac{2}{x^2}}{5+\frac{1}{x^2}} \;=\; 0$$

Eine weitere Lösungsvariante wird hier gezeigt:

geg.: $f(x)=\frac{2x-1}{5x^2+1}\sqrt{3+\sin 2x}$

$$\sqrt{2} \;\le\; \sqrt{3+\underbrace{\sin 2x}_{-1\le y\le 1}} \;\le\; 2$$

$$g(x) \;\le\; f(x) \;\le\; h(x)$$

bzw.

$$\lim_{x\to\infty} g(x) \;\le\; \lim_{x\to\infty} f(x) \;\le\; \lim_{x\to\infty} h(x)$$

$$\lim_{x\to\infty}\frac{2x-1}{5x^2+1}\cdot\sqrt{2} \;\le\; \lim_{x\to\infty} f(x) \;\le\; \lim_{x\to\infty}\frac{2x-1}{5x^2+1}\cdot 2$$

$$\sqrt{2}\cdot\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^2}}{5+\frac{1}{x^2}} \;\le\; \lim_{x\to\infty} f(x) \;\le\; 2\cdot\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^2}}{5+\frac{1}{x^2}}$$

$$\sqrt{2}\cdot 0 \;\le\; \lim_{x\to\infty} f(x) \;\le\; 2\cdot 0$$

$$\lim_{x\to\infty} f(x) \;=0$$