Verschiedene Stegreifaufgaben

Klasse 9 / I / II

- Lösungen -

1.1 $1+\sqrt{2}$, $3,\overline{10}$, π , $\sqrt{10}$, $\sqrt{\sqrt{101}}$, $\frac{10}{3}$

1.2

a) $\sqrt{\frac{75}{12}} = \sqrt{\frac{3\cdot 25}{3\cdot 4}} = \frac{5}{2} = \underline{\underline{2,5}}$

b) $\sqrt{98a^2b^2} = ab\sqrt{2\cdot 49} = \underline{\underline{7ab\sqrt{2}}}$

c) $\sqrt{3u^2+6uv+3v^2} = \sqrt{3(u+v)^2} = \underline{\underline{(u+v)\sqrt{3}}}$

1.3

a) $\frac{4-\sqrt{20}}{4\sqrt{5}} = \frac{(4-\sqrt{20})\cdot\sqrt{5}}{4\sqrt{5}\cdot\sqrt{5}} = \frac{4\sqrt{5}-\sqrt{100}}{20} = \underline{\underline{0,2\sqrt{5}-0,5}}$

b) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{2+2\sqrt{6}+3}{2-3} = \underline{\underline{-(5+2\sqrt{6})}}$

1.4

a) $(3\sqrt{27}-2\sqrt{12})\cdot(-\sqrt{3}) = -3\sqrt{81}+2\sqrt{36} = -27+12 = \underline{\underline{-15}}$

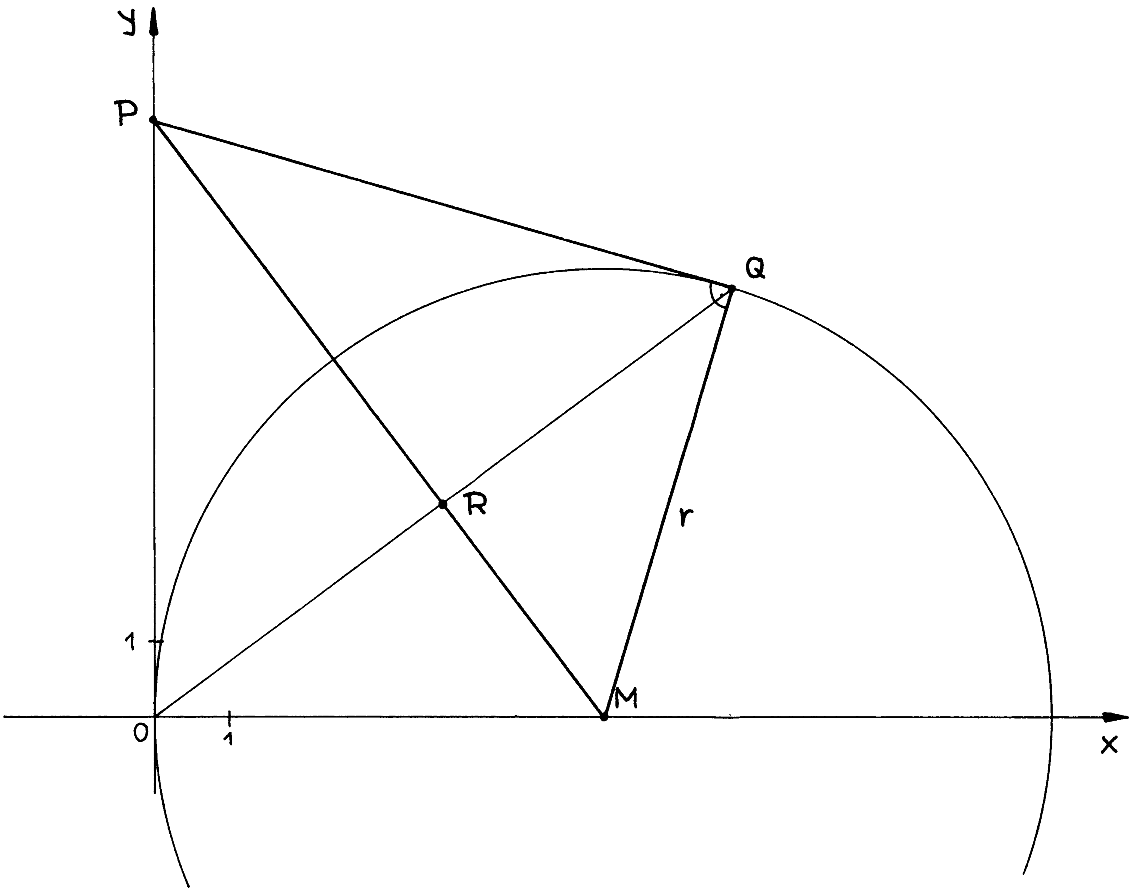
b) $(2\sqrt{5}+5\sqrt{3})(2\sqrt{5}-5\sqrt{3}) = 4\cdot 5 - 25\cdot 3 = \underline{\underline{-55}}$

c) $(\sqrt{6}-\sqrt{3})^2 = 6-2\sqrt{18}+3 = 9-2\sqrt{2\cdot 9} = \underline{\underline{9-6\sqrt{2}}}$

d) $\frac{\sqrt{14}}{\sqrt{3}}\cdot\frac{\sqrt{7}}{\sqrt{3}} + \frac{\sqrt{21}}{\sqrt{27}}\cdot\frac{\sqrt{7}}{\sqrt{3}} = \frac{7\sqrt{2}}{3} + \frac{7\sqrt{3}}{9} = \underline{\underline{\frac{7}{3}\left(\sqrt{2}+\frac{1}{3}\sqrt{3}\right)}}$

- Lösungen -

2.1



2.2

$\overline{MP} = \sqrt{6^2 + 8^2}$ Pythagoras

$\overline{MP} = 10 \text{ cm}$

2.3

wenn ΔPMQ rechtwinklig, gilt Satz d. Pythagoras:

$\overline{MP}^2 = \overline{PQ}^2 + \overline{MQ}^2$

$100 = 64 + 36$

$100 = 100 \ (w)$

ΔPMQ ist bei Q rechtwinklig.

$\overline{MQ}$: $MQ^2 = (x_Q - x_M)^2 + y_Q^2$

$\overline{MQ}^2 = (7{,}68 - 6)^2 + 5{,}76^2$

$\overline{MQ}^2 = 36 \text{ cm}$

$\overline{PQ}^2 = x_Q + (y_P - y_Q)^2$

$\overline{PQ}^2 = 7{,}68^2 + (8 - 5{,}76)^2$

$\overline{PQ}^2 = 64 \text{cm}$

2.4

Kathetensatz: $\overline{MQ}^2 = \overline{MR} \cdot \overline{MP}$

$\overline{MR} = \frac{36}{10}$

$\overline{MR} = 3{,}6 \text{ cm}$

Pythagoras: $\overline{RQ}^2 = \overline{MQ}^2 - \overline{MR}^2$

$\overline{RQ} = \sqrt{36 - 12{,}96}$

$\overline{RQ} = 4{,}8 \text{ cm}$

- Lösungen -

3.2

$$\overrightarrow{AB} = \begin{pmatrix} 2+4 \\ -3+1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

$$\overrightarrow{AD} = \begin{pmatrix} -1+4 \\ 3+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

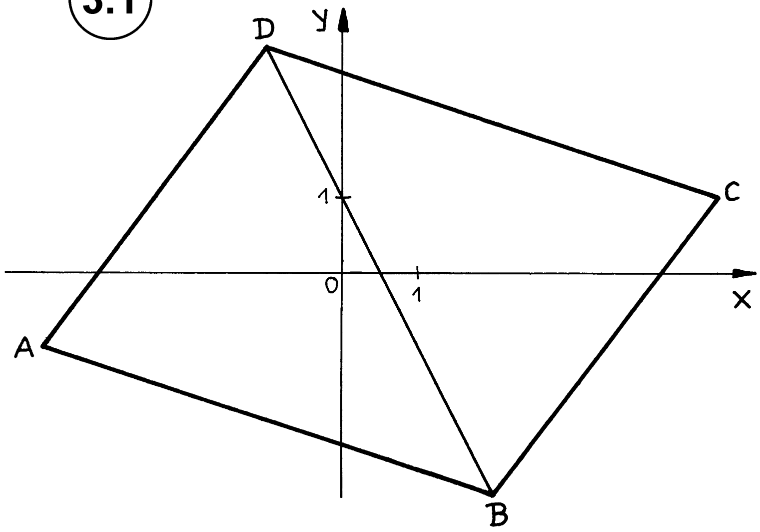
$$A_{ABCD} = \left| \overrightarrow{AB} \ \ \overrightarrow{AD} \right| \text{ FE}$$

$$A_{ABCD} = \begin{vmatrix} 6 & 3 \\ -2 & 4 \end{vmatrix} \text{ FE}$$

$$A_{ABCD} = (24+6) \text{ FE}$$

$$\underline{\underline{A_{ABCD} = 30 \text{ FE}}}$$

3.1



3.3

$$\overrightarrow{BC} = \overrightarrow{AD}$$

$$\begin{pmatrix} x-2 \\ y+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Longrightarrow \begin{matrix} x = 5 \\ y = 1 \end{matrix} \Longrightarrow \underline{\underline{C(5|1)}}$$

4.2

$$A = A_{\Delta STU} + A_{\Delta SUV}$$

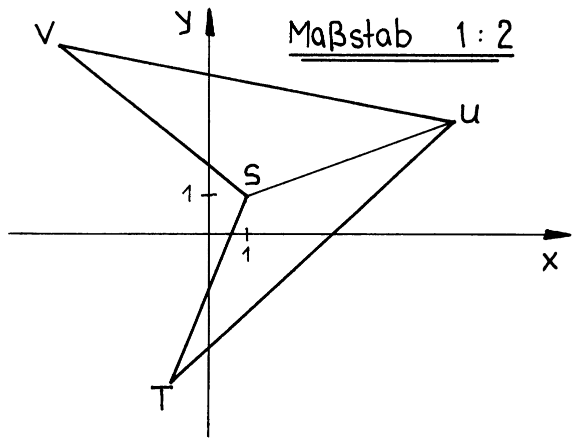
$$A = \tfrac{1}{2}\left|\overrightarrow{ST} \ \ \overrightarrow{SU}\right| + \tfrac{1}{2}\left|\overrightarrow{SU} \ \ \overrightarrow{SV}\right| \text{ FE}$$

$$A = \tfrac{1}{2}\begin{vmatrix} -2 & 5{,}5 \\ -5 & 2 \end{vmatrix} + \tfrac{1}{2}\begin{vmatrix} 5{,}5 & -5 \\ 2 & 4 \end{vmatrix} \text{ FE}$$

$$A = \tfrac{1}{2}(-4+27{,}5) + \tfrac{1}{2}(22+10) \text{ FE}$$

$$\underline{\underline{A = 27{,}75 \text{ FE}}}$$

4.1



$$\overrightarrow{ST} = \begin{pmatrix} -1-1 \\ -4-1 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

$$\overrightarrow{SU} = \begin{pmatrix} 6{,}5-1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 5{,}5 \\ 2 \end{pmatrix}$$

$$\overrightarrow{SV} = \begin{pmatrix} -4-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$

- Lösungen -

5.1 Vierstreckensatz: $\dfrac{h-a}{e_2} = \dfrac{2m-a}{e_1}$

$$\underline{\underline{h = \frac{e_2(2-a)}{e_1} + a}}$$

5.2

$$h = \frac{23{,}75m \cdot (2m - 1{,}60m)}{1{,}25m} + 1{,}60m$$

$$\underline{\underline{h = 9{,}20m}}$$

6.1

$\overline{Q_nR_n} : \overline{AB} = (\overline{CP} - h_n) : \overline{CP}$ (Vierstreckensatz)

$$\frac{\overline{Q_nR_n}}{12{,}5} = \frac{5-h_n}{5}$$

$$\overline{Q_nR_n} = 2{,}5\,(5-h_n)$$

$$\underline{\underline{\overline{Q_nR_n} = -2{,}5\,h_n + 12{,}5}}$$

6.2

$$A_{(h)} = \frac{1}{2} \cdot g \cdot h$$

$$A_{(h)} = \frac{1}{2} \cdot \overline{Q_nR_n} \cdot h_n$$

$$A_{(h)} = \frac{1}{2} \cdot h_n \cdot (-2{,}5\,h_n + 12{,}5)$$

$$A_{(h)} = -1{,}25\,h_n^2 + 6{,}25\,h_n$$

$$A_{(h)} = -1{,}25\,(h_n^2 - 5\,h_n)$$

$$A_{(h)} = -1{,}25\,(h_n^2 - 5\,h_n + 2{,}5^2 - 2{,}5^2)$$

$$A_{(h)} = -1{,}25\,[(h_n - 2{,}5)^2 - 6{,}25]$$

$$\underline{A_{(hn)} = -1{,}25\,(h_n - 2{,}5)^2 + 7{,}8125}$$

$$\underline{\underline{A_{max} = 7{,}81\ \text{FE} \quad \text{für } h = 2{,}5}}$$

- Lösungen -

7.1

$y = m \cdot x$ A(-7|3) einsetzen

$3 = m \cdot (-7)$

$\underline{m = -\frac{3}{7}}$

$\underline{\underline{y = -\frac{3}{7}x}}$

7.2

$y = m \cdot x$ $B(-\frac{2}{5}|-\frac{5}{4})$ einsetzen

$-\frac{5}{4} = m \cdot (-\frac{2}{5})$

$\underline{m = \frac{25}{8}}$

$\underline{\underline{y = \frac{25}{8}x}}$

8.

$A_1(6|9)$ einsetzen in $y = mx$: $9 = m \cdot 6$

$\underline{m = 1{,}5}$

$A_2(-3|-4{,}8)$ einsetzen $-4{,}8 = m \cdot (-3)$

$\underline{m = 1{,}6}$

Da die Steigungen nicht gleich sind, liegen A_1 und A_2 nicht auf derselben Ursprungsgeraden.

9.1

$3y - 4{,}5x = 0$

$3y = 4{,}5x \quad |:3$

g_1: $\underline{\underline{y = 1{,}5x}}$ h_1: $\underline{\underline{y = -\frac{2}{3}x}}$

9.2

$\frac{3}{2}x - 6y = 0$

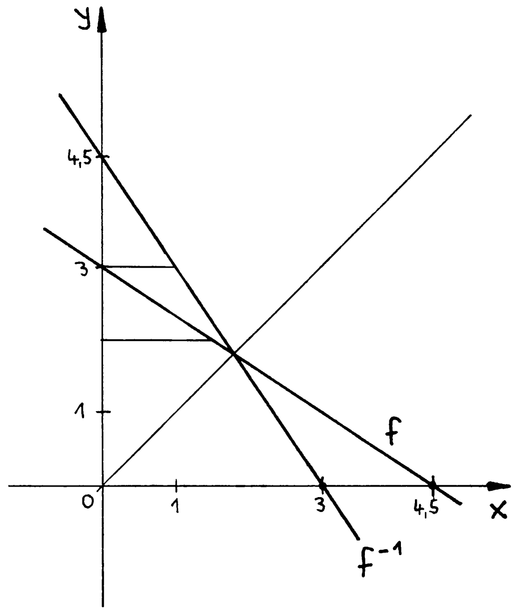
$-6y = -\frac{3}{2}x \quad |:(-6)$

g_2: $\underline{\underline{y = \frac{1}{4}x}}$ h_2: $\underline{\underline{y = -4x}}$

- Lösungen -

10.1

$$-3x - 4{,}5\,y + 13{,}5 = 0$$
$$-4{,}5\,y = 3x - 13{,}5 \quad /:(-4{,}5)$$
$$f: \quad \underline{y = -\tfrac{2}{3}x + 3}$$



$\underline{y = 0:}$
$$0 = -\tfrac{2}{3}x_0 + 3$$
$$\tfrac{2}{3}x_0 = 3 \quad /\cdot\tfrac{3}{2}$$

Nullstelle von f: $\underline{\underline{x_0 = 4{,}5}}$

f^{-1}:
$$x = -\tfrac{2}{3}y + 3$$
$$\tfrac{2}{3}y = -x + 3 \quad /\cdot\tfrac{3}{2}$$
$$f^{-1}: \quad \underline{\underline{y = -\tfrac{3}{2}x + 4{,}5}}$$

$\underline{y = 0:}$
$$0 = -\tfrac{3}{2}x_0 + 4{,}5$$
$$\tfrac{3}{2}x_0 = 4{,}5$$

Nullstelle von f^{-1}: $\underline{\underline{x_0 = 3}}$

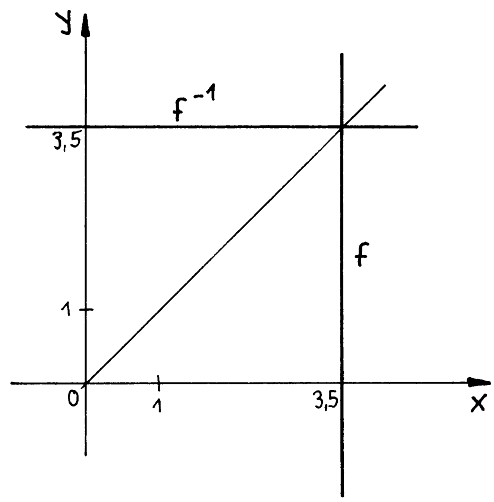
Schnittpunkte mit x-Achse:

$\underline{\underline{f: \; S(4{,}5|0)}}$

$\underline{\underline{f^{-1}: \; S(3|0)}}$

10.2

$$x - 3{,}5 = 0$$
$$f: \quad \underline{x = 3{,}5}$$
$$f^{-1}: \quad \underline{\underline{y = 3{,}5}}$$



Nullstellen: f: $x_0 = 3{,}5$

f^{-1}: keine Nullstelle

Schnittpunkte mit x-Achse:

f: $S(3{,}5|0)$

f^{-1}: kein Schnittpunkt

- Lösungen -

11.1

Koordinaten von P u. Q in $y = ax^2 + 2x + c$ einsetzen:

$$\left| \begin{array}{l} 2 = a \cdot 0^2 + 2 \cdot 0 + c \\ \wedge -1 = a \cdot 3^2 + 2 \cdot 3 + c \end{array} \right.$$

$$\left| \begin{array}{l} 2 = c \\ \wedge -1 = 9a + 6 + c \end{array} \right.$$

$$\left| \begin{array}{l} 2 = c \\ \wedge\ 9a = -7 - c \end{array} \right.$$

$$9a = -7 - 2$$
$$\underline{a = -1}$$

$\underline{b = 2}$

$\underline{c = 2}$

$\underline{a = -1}$

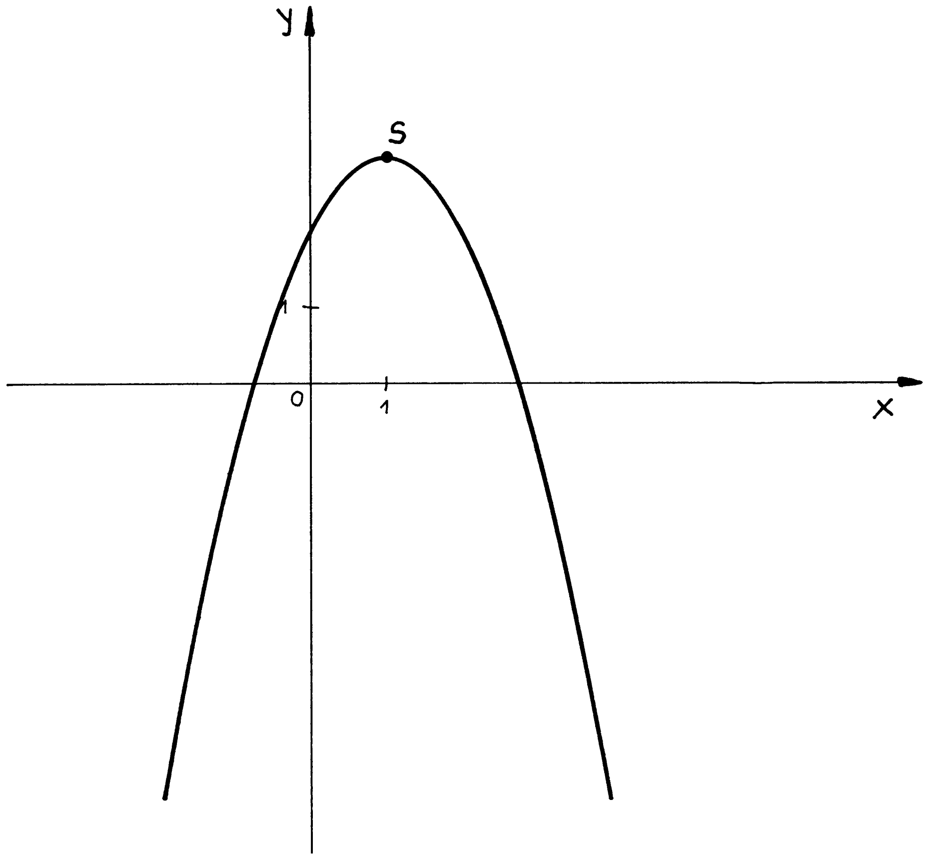
$$\underline{\underline{y = -x^2 + 2x + 2}}$$

11.2

$$y = -(x^2 - 2x - 2)$$
$$y = -(x^2 - 2x + 1^2 - 1 - 2)$$
$$y = -[(x-1)^2 - 3]$$
$$\underline{y = -(x-1)^2 + 3}$$

$$\underline{\underline{S(1 \mid 3)}}$$

11.3



- Lösungen -

12. Koordinaten von P:

$P(0|1)$

Abbildung von P:

$P(0|1) \xmapsto{Z(2|1)\,;\,k=-1} P'$

$\overrightarrow{ZP'} = k \cdot \overrightarrow{ZP}$

$\begin{pmatrix} x'-2 \\ y'-1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 0-2 \\ 1-1 \end{pmatrix}$

$\begin{vmatrix} x' = 4 \\ \wedge\ y' = 1 \end{vmatrix}$

$\underline{P'(4|1)}$

Punktsteigungsform von g':

$y = m\,(x - x_{p}') + y_{p}'$

Koordinaten von P' und Steigung m = 1
einsetzen (Steigung ist bei g und g' gleich):

$y = 1\,(x-4) + 1$

g': $\underline{\underline{y = x - 3}}$

- Lösungen -

13.

Koordinaten von P:

$y_P = 2^2$

$\underline{y_P = 4}$ $\Longrightarrow$ $\underline{P(2|4)}$

Scheitelkoordinaten:

$\underline{S(0|0)}$

Abbildung von P:

$P(2|4) \xmapsto{Z(2|1);\ k=-3} P'$

$\overrightarrow{ZP'} = k \cdot \overrightarrow{ZP}$

$\begin{pmatrix} x'-2 \\ y'-1 \end{pmatrix} = -3 \cdot \begin{pmatrix} 2-2 \\ 4-1 \end{pmatrix}$

$\left| \begin{array}{l} x' = 2 \\ \wedge\ y' = -2 \end{array} \right.$

$\underline{P'(2|-8)}$

Abbildung von S:

$S(0|0) \xmapsto{Z(2|1);\ k=-3} S'$

$\overrightarrow{ZS'} = k \cdot \overrightarrow{ZS}$

$\begin{pmatrix} x'-2 \\ y'-1 \end{pmatrix} = -3 \cdot \begin{pmatrix} 0-2 \\ 0-1 \end{pmatrix}$

$\left| \begin{array}{l} x' = 8 \\ \wedge\ y' = 4 \end{array} \right.$

$\underline{S'(8|4)}$

Scheitelgleichung der Bildparabel:

$y = a(x - x_s')^2 + y_s'$

$\underline{y = a(x-8)^2 + 4}$

Koordinaten von P' einsetzen:

$-8 = a(2-8)^2 + 4$

$a = \frac{-12}{36}$

$\underline{a = -\frac{1}{3}}$

Gleichung der Bildparabel:

$p': \underline{\underline{y = -\frac{1}{3}(x-8)^2 + 4}}$