

Verschiedene Stegreifaufgaben

Klasse 9 / I / II

- Lösungen -

1.1

$$1+\sqrt{2}, \quad 3,\overline{10}, \quad \pi, \quad \sqrt{10}, \quad \sqrt{\sqrt{101}}, \quad \frac{10}{3}$$

1.2

$$a), \quad \sqrt{\frac{75}{12}} = \sqrt{\frac{3 \cdot 25}{3 \cdot 4}} = \frac{5}{2} = \underline{\underline{2,5}}$$

$$b), \quad \sqrt{98a^2b^2} = ab\sqrt{2 \cdot 49} = \underline{\underline{7ab\sqrt{2}}}$$

$$c), \quad \sqrt{3u^2 + 6uv + 3v^2} = \sqrt{3(u+v)^2} = \underline{\underline{(u+v)\sqrt{3}}}$$

1.3

$$a), \quad \frac{4 - \sqrt{20}}{4\sqrt{5}} = \frac{(4 - \sqrt{20}) \cdot \sqrt{5}}{4\sqrt{5} \cdot \sqrt{5}} = \frac{4\sqrt{5} - \sqrt{100}}{20} = \underline{\underline{0,2\sqrt{5} - 0,5}}$$

$$b), \quad \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})} = \frac{2 + 2\sqrt{6} + 3}{2 - 3} = \underline{\underline{-(5 + 2\sqrt{6})}}$$

1.4

$$a), \quad (3\sqrt{27} - 2\sqrt{12}) \cdot (-\sqrt{3}) = -3\sqrt{81} + 2\sqrt{36} = -27 + 12 = \underline{\underline{-15}}$$

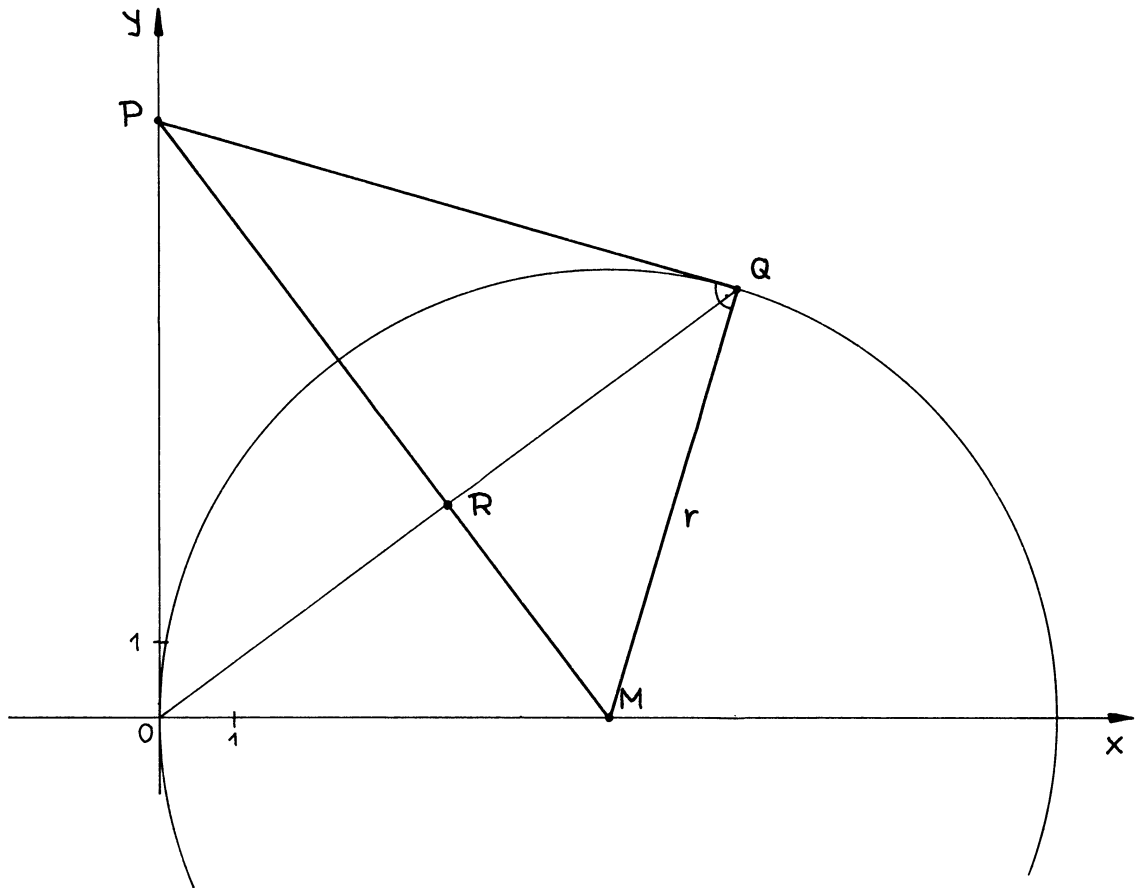
$$b), \quad (2\sqrt{5} + 5\sqrt{3})(2\sqrt{5} - 5\sqrt{3}) = 4 \cdot 5 - 25 \cdot 3 = \underline{\underline{-55}}$$

$$c), \quad (\sqrt{6} - \sqrt{3})^2 = 6 - 2\sqrt{18} + 3 = 9 - 2\sqrt{2 \cdot 9} = \underline{\underline{9 - 6\sqrt{2}}}$$

$$d), \quad \frac{\sqrt{14}}{\sqrt{3}} \cdot \frac{\sqrt{7}}{\sqrt{3}} + \frac{\sqrt{21}}{\sqrt{27}} \cdot \frac{\sqrt{7}}{\sqrt{3}} = \frac{7\sqrt{2}}{3} + \frac{7\sqrt{3}}{9} = \underline{\underline{\frac{7}{3}(\sqrt{2} + \frac{1}{3}\sqrt{3})}}$$

- Lösungen -

2.1



2.2

$$\overline{MP} = \sqrt{6^2 + 8^2} \quad \text{Pythagoras}$$

$$\underline{\underline{\overline{MP} = 10 \text{ cm}}}$$

2.3

wenn $\triangle PMQ$ rechtwinklig, gilt Satz d. Pythagoras:

$$\overline{MP}^2 = \overline{PQ}^2 + \overline{MQ}^2 \quad \overline{MQ}: \quad \overline{MQ}^2 = (x_Q - x_M)^2 + y_Q^2$$

$$100 = 64 + 36$$

$$\overline{MQ}^2 = (7,68 - 6)^2 + 5,76^2$$

$$\underline{100 = 100 \text{ (w)}}$$

$$\underline{\overline{MQ}^2 = 36 \text{ cm}}$$

$\triangle PMQ$ ist bei Q
rechtwinklig.

$$\overline{PQ}^2 = x_Q + (y_P - y_Q)^2$$

$$\overline{PQ}^2 = 7,68^2 + (8 - 5,76)^2$$

$$\underline{\overline{PQ}^2 = 64 \text{ cm}}$$

2.4

$$\text{Kathetensatz: } \overline{MQ}^2 = \overline{MR} \cdot \overline{MP} \quad \text{Pythagoras: } \overline{RQ}^2 = \overline{MQ}^2 - \overline{MR}^2$$

$$\overline{MR} = \frac{36}{10}$$

$$\underline{\underline{\overline{MR} = 3,6 \text{ cm}}}$$

$$\overline{RQ} = \sqrt{36 - 12,96}$$

$$\underline{\underline{\overline{RQ} = 4,8 \text{ cm}}}$$

- Lösungen -

3.2

$$\vec{AB} = \begin{pmatrix} 2+4 \\ -3+1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

$$\vec{AD} = \begin{pmatrix} -1+4 \\ 3+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

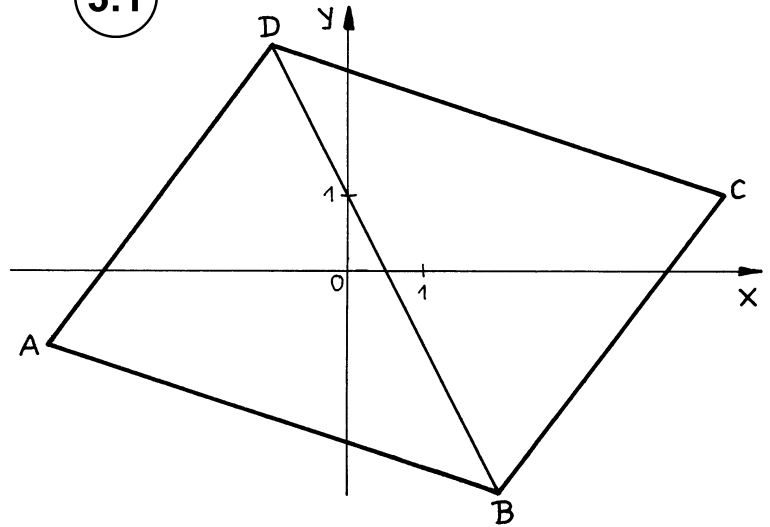
$$A_{ABCD} = |\vec{AB} \ \vec{AD}| \text{ FE}$$

$$A_{ABCD} = \begin{vmatrix} 6 & 3 \\ -2 & 4 \end{vmatrix} \text{ FE}$$

$$A_{ABCD} = (24 + 6) \text{ FE}$$

$$\underline{\underline{A_{ABCD} = 30 \text{ FE}}}$$

3.1



3.3

$$\vec{BC} = \vec{AD}$$

$$\begin{pmatrix} x-2 \\ y+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \implies \begin{matrix} x=5 \\ y=1 \end{matrix} \implies \underline{\underline{C(5|1)}}$$

4.2

$$A = A_{\Delta STU} + A_{\Delta SUV}$$

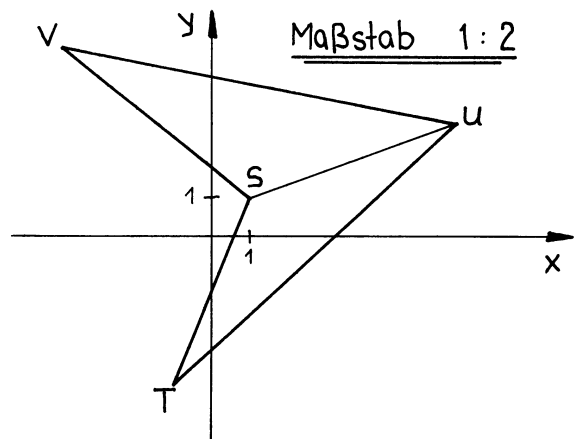
$$A = \frac{1}{2} |\vec{ST} \ \vec{SU}| + \frac{1}{2} |\vec{SU} \ \vec{SV}| \text{ FE}$$

$$A = \frac{1}{2} \begin{vmatrix} -2 & 5,5 \\ -5 & 2 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} 5,5 & -5 \\ 2 & 4 \end{vmatrix} \text{ FE}$$

$$A = \frac{1}{2} (-4 + 27,5) + \frac{1}{2} (22 + 10) \text{ FE}$$

$$\underline{\underline{A = 27,75 \text{ FE}}}$$

4.1



$$\vec{ST} = \begin{pmatrix} -1 & -1 \\ -4 & -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

$$\vec{SU} = \begin{pmatrix} 6,5 & -1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 5,5 \\ 2 \end{pmatrix}$$

$$\vec{SV} = \begin{pmatrix} -4 & -1 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$

- Lösungen -

5.1 Vierstreckensatz : $\frac{h-a}{e_2} = \frac{2m-a}{e_1}$

$$h = \frac{e_2(2-a)}{e_1} + a$$

5.2

$$h = \frac{23,75m \cdot (2m - 1,60m)}{1,25m} + 1,60m$$
$$h = \underline{\underline{9,20m}}$$

6.1 $\overline{Q_n R_n} : \overline{AB} = (\overline{CP} - h_n) : \overline{CP}$ (Vierstreckensatz)

$$\frac{\overline{Q_n R_n}}{12,5} = \frac{5 - h_n}{5}$$
$$\overline{Q_n R_n} = 2,5(5 - h_n)$$
$$\underline{\underline{\overline{Q_n R_n} = -2,5 h_n + 12,5}}$$

6.2

$$A(h) = \frac{1}{2} \cdot g \cdot h$$
$$A(h) = \frac{1}{2} \cdot \overline{Q_n R_n} \cdot h_n$$
$$A(h) = \frac{1}{2} \cdot h_n \cdot (-2,5 h_n + 12,5)$$
$$A(h) = -1,25 h_n^2 + 6,25 h_n$$
$$A(h) = -1,25 (h_n^2 - 5 h_n)$$
$$A(h) = -1,25 (h_n^2 - 5 h_n + 2,5^2 - 2,5^2)$$
$$A(h) = -1,25 [(h_n - 2,5)^2 - 6,25]$$
$$\underline{\underline{A(h_n) = -1,25 (h_n - 2,5)^2 + 7,8125}}$$

$$\underline{\underline{A_{\max} = 7,81 \text{ FE für } h = 2,5}}$$

- Lösungen -

7.1

$$\begin{aligned} y &= m \cdot x && A(-7|3) \text{ einsetzen} \\ 3 &= m \cdot (-7) \\ m &= \underline{\underline{-\frac{3}{7}}} \\ y &= \underline{\underline{-\frac{3}{7}x}} \end{aligned}$$

7.2

$$\begin{aligned} y &= m \cdot x && B(-\frac{2}{5} | -\frac{5}{4}) \text{ einsetzen} \\ -\frac{5}{4} &= m \cdot (-\frac{2}{5}) \\ m &= \underline{\underline{\frac{25}{8}}} \\ y &= \underline{\underline{\frac{25}{8}x}} \end{aligned}$$

8.

$$A_1(6|9) \text{ einsetzen in } y = mx: \quad \begin{aligned} 9 &= m \cdot 6 \\ m &= \underline{\underline{1,5}} \end{aligned}$$

$$A_2(-3|-4,8) \text{ einsetzen} \quad \begin{aligned} -4,8 &= m \cdot (-3) \\ m &= \underline{\underline{1,6}} \end{aligned}$$

Da die Steigungen nicht gleich sind, liegen A_1 und A_2 nicht auf derselben Ursprungsgeraden.

9.1

$$\begin{aligned} 3y - 4,5x &= 0 \\ 3y &= 4,5x \quad /:3 \\ g_1: \quad y &= \underline{\underline{1,5x}} && h_1: \quad \underline{\underline{y = -\frac{2}{3}x}} \end{aligned}$$

9.2

$$\begin{aligned} \frac{3}{2}x - 6y &= 0 \\ -6y &= -\frac{3}{2}x \quad /:(-6) \\ g_2: \quad y &= \underline{\underline{\frac{1}{4}x}} && h_2: \quad \underline{\underline{y = -4x}} \end{aligned}$$

- Lösungen -

10.1

$$-3x - 4,5y + 13,5 = 0$$

$$-4,5y = 3x - 13,5 \quad | :(-4,5)$$

$$f: \quad \underline{y = -\frac{2}{3}x + 3}$$

$$\underline{y=0}: \quad 0 = -\frac{2}{3}x_0 + 3$$

$$\frac{2}{3}x_0 = 3 \quad | \cdot \frac{3}{2}$$

$$\text{Nullstelle von } f: \quad \underline{\underline{x_0 = 4,5}}$$

$$f^{-1}: \quad x = -\frac{2}{3}y + 3$$

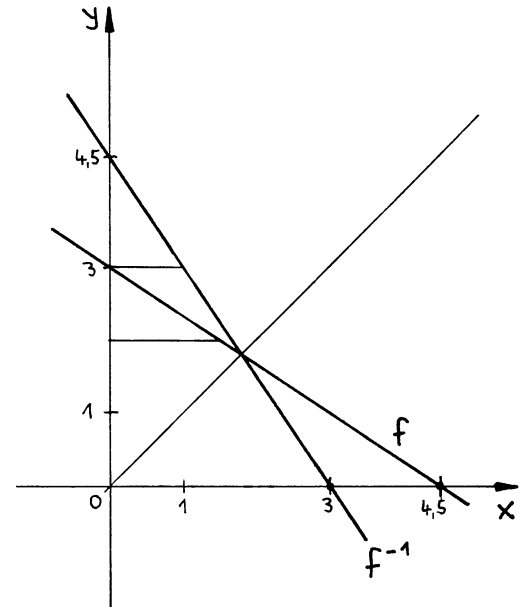
$$\frac{2}{3}y = -x + 3 \quad | \cdot \frac{3}{2}$$

$$f^{-1}: \quad \underline{\underline{y = -\frac{3}{2}x + 4,5}}$$

$$\underline{y=0}: \quad 0 = -\frac{3}{2}x_0 + 4,5$$

$$\frac{3}{2}x_0 = 4,5$$

$$\text{Nullstelle von } f^{-1}: \quad \underline{\underline{x_0 = 3}}$$



Schnittpunkte mit x-Achse:

$$\underline{\underline{f: S(4,5|0)}}$$

$$\underline{\underline{f^{-1}: S(3|0)}}$$

10.2

$$f: \quad \underline{x - 3,5 = 0}$$

$$f^{-1}: \quad \underline{\underline{y = 3,5}}$$

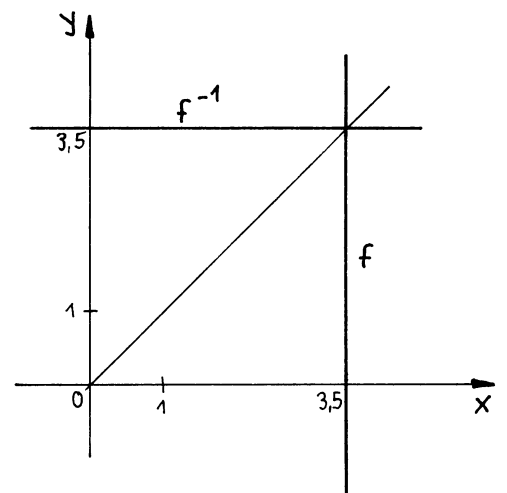
$$\text{Nullstellen: } f: x_0 = 3,5$$

$$f^{-1}: \text{keine Nullstelle}$$

Schnittpunkte mit x-Achse:

$$f: S(3,5|0)$$

$$f^{-1}: \text{kein Schnittpunkt}$$



- Lösungen -

11.1

Koordinaten von P u. Q in $y = ax^2 + 2x + c$ einsetzen:

$$2 = a \cdot 0^2 + 2 \cdot 0 + c$$

$$\wedge -1 = a \cdot 3^2 + 2 \cdot 3 + c$$

$$\underline{b = 2}$$

$$\begin{array}{l} 2 = c \\ \wedge -1 = 9a + 6 + c \end{array}$$

$$\underline{c = 2}$$

$$\underline{a = -1}$$

$$\begin{array}{l} 2 = c \\ \wedge 9a = -7 - c \end{array}$$

$$9a = -7 - 2$$

$$\underline{a = -1}$$

$$\underline{\underline{y = -x^2 + 2x + 2}}$$

11.2

$$y = -(x^2 - 2x - 2)$$

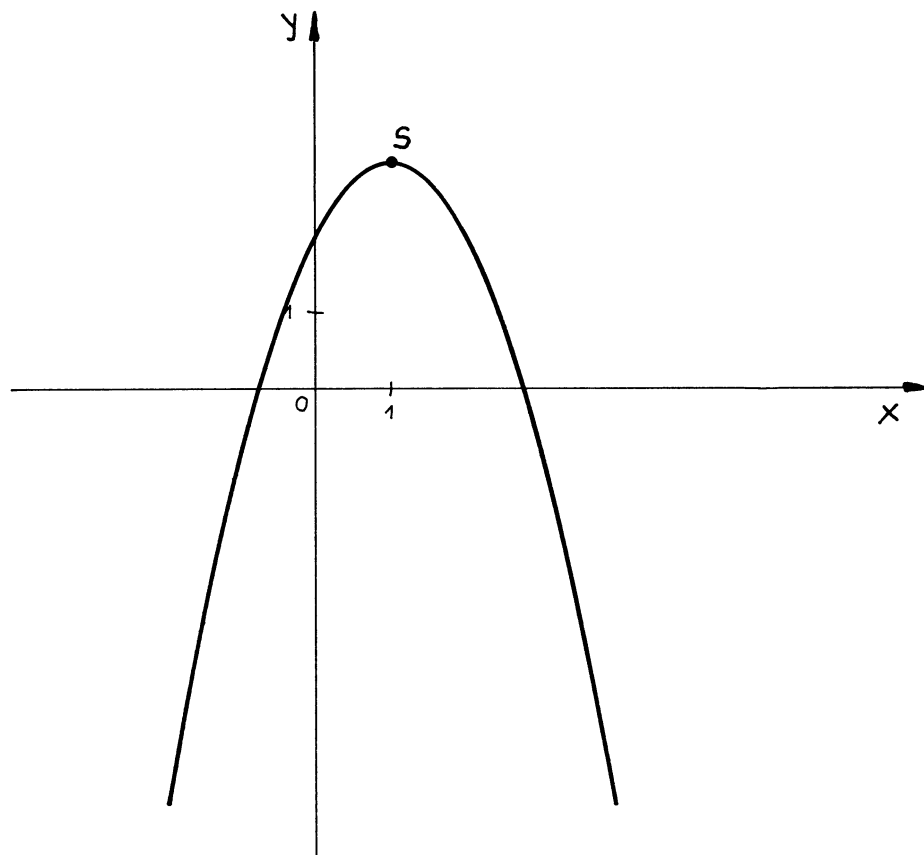
$$y = -(x^2 - 2x + 1^2 - 1 - 2)$$

$$y = -[(x-1)^2 - 3]$$

$$\underline{y = -(x-1)^2 + 3}$$

$$\underline{\underline{S(1|3)}}$$

11.3



- Lösungen -

12.

Koordinaten von P:

$$P(0|1)$$

Abbildung von P:

$$P(0|1) \xrightarrow{Z(2|1); k=-1} P'$$

$$\vec{ZP'} = k \cdot \vec{ZP}$$

$$\begin{pmatrix} x' - 2 \\ y' - 1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 0 - 2 \\ 1 - 1 \end{pmatrix}$$

$$\begin{cases} x' = 4 \\ \wedge y' = 1 \end{cases}$$

$$\underline{P'(4|1)}$$

Punktsteigungsform von g' :

$$y = m(x - x_p') + y_p'$$

Koordinaten von P' und Steigung $m=1$
einsetzen (Steigung ist bei g und g' gleich):

$$y = 1(x - 4) + 1$$

$$g': \underline{\underline{y = x - 3}}$$

- Lösungen -

13.

Koordinaten von P:

$$\begin{aligned} y_P &= 2^2 \\ y_P &= 4 \quad \Rightarrow \quad \underline{P(2|4)} \end{aligned}$$

Scheitelkoordinaten:

$$\underline{S(0|0)}$$

Abbildung von P:

$$P(2|4) \xrightarrow{Z(2|1); k=-3} P'$$

$$\vec{ZP'} = k \cdot \vec{ZP}$$

$$\begin{pmatrix} x'-2 \\ y'-4 \end{pmatrix} = -3 \cdot \begin{pmatrix} 2-2 \\ 4-1 \end{pmatrix}$$

$$\begin{cases} x' = 2 \\ \wedge y' = -2 \end{cases}$$

$$\underline{P'(2|-2)}$$

Abbildung von S:

$$S(0|0) \xrightarrow{Z(2|1); k=-3} S'$$

$$\vec{ZS'} = k \cdot \vec{ZS}$$

$$\begin{pmatrix} x'-2 \\ y'-0 \end{pmatrix} = -3 \cdot \begin{pmatrix} 0-2 \\ 0-1 \end{pmatrix}$$

$$\begin{cases} x' = 8 \\ \wedge y' = 4 \end{cases}$$

$$\underline{S'(8|4)}$$

Scheitelform der Bildparabel:

$$y = a(x - x_{s'})^2 + y_{s'}$$

$$\underline{y = a(x - 8)^2 + 4}$$

Koordinaten von P' einsetzen:

$$-2 = a(2 - 8)^2 + 4$$

$$a = \frac{-12}{36}$$

$$\underline{a = -\frac{1}{3}}$$

Gleichung der Bildparabel:

$$P': \underline{\underline{y = -\frac{1}{3}(x - 8)^2 + 4}}$$